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In 2006 all ECB publications feature a motif taken from the €5 banknote.

1 The views expressed in this paper are those of the author and do not necessarily represent those of the European Central Bank. I am grateful to C. Osbat and L. Sarno for technical advice on the econometric.

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Abstract: The paper tests for nonlinearities in the adjustment of the euro exchange rate towards purchasing power parity (PPP). It presents new survey based evidence consistent with non-linear patterns in euro exchange rate dynamics. Moreover, based on an exponential smooth transition autoregressive (ESTAR-) model, it finds strong evidence that the speed of mean reversion in euro exchange rates increases non-linearly with the magnitude of the PPP deviation. Accordingly, while the euro real exchange rate can be well approximated by a random walk if PPP deviations are small, in periods of significant deviations, gravitational forces are set to take root and bring the exchange rate back towards its long-term trend. Consistent with higher euro-dollar volatility, deviations from the PPP equilibrium for this pair need to be stronger in order to reach the same adjustment intensity as for other currencies.

JEL: F31

Keywords: Purchasing power parity, real exchange rate, non-linearities, STAR models

Non-technical summary

The wide fluctuations in the euro exchange rate since its launch in 1999 call for a deeper understanding of the underlying relationships. From a policy perspective, it is particularly useful to understand of whether and when exchange rate changes constitute movements towards or away from some long-term equilibrium level. While there are various ways to think about equilibrium exchange rates, this paper focuses on the universal starting point of applied exchange rate analysis which is to compare the current exchange rate against some measure of purchasing power parity (PPP). In its relative version, PPP states that nominal exchange rates move in proportion to relative developments in domestic and foreign prices implying a constant real exchange rate. While visual inspection seems to support that the euro real exchange rates oscillate around a mean in the long run, more rigorous statistical tests commonly fail to confirm such mean reverting properties.

This finding should not lead us to the conclusion to discard drawing inferences from PPP analysis. Several caveats caution against a too strict interpretation of such tests of PPP. These include the lack of power of the tests themselves, the real exchange rate interaction with economic fundamentals (such as different trends in productivity) and the presence of non-linearities in the mean reversion process. This paper investigates the latter aspect.

Non-linearities in the adjustment of exchange rates can be justified from both a goods and a financial market perspective. As international goods arbitrage involves transaction costs, arbitrage sets in once the real exchange rate protractedly moves outside certain limits. From an asset market perspective, differing trading strategies could play a role with "Chartists" dominating market dynamics when the exchange rate is close to some perceived equilibrium, while "Fundamentalists" being at play once the exchange rate is increasingly misaligned.

Appling this concept to the euro, the presence of such non-linear behaviour is confirmed for the real effective exchange rate of the euro and the two bilateral euro exchange rates with the biggest weight in the euro effective exchange rate, namely the real euro-dollar and the real euro-pound sterling exchange rates. Specifically, using a smooth transition autoregressive (STAR-) model, we find that the further away the real exchange rate moves from PPP, the stronger the adjustment intensity becomes. Our estimates suggest that in past episodes of large PPP deviations of the euro exchange rate – such as in the mid-1980s and in late 2000 – the implied half-life of adjustment to PPP, which is a standard measure of how long it takes for the effect of a shock to die out, declined to just 2-3 quarters. This compares to an estimated average mean reverting speed to PPP of three to five years.

1. Introduction

The wide fluctuations in the euro exchange rate since its launch in 1999 call for a deeper understanding of the basic underlying relationships. Between the January 1999 and the October 2000, the euro depreciated by around 20% in nominal effective terms. This reflected a significant weakening of the euro against the currencies of major trading partners such as the US dollar (-26%), the pound sterling (-16%) and the Japanese yen (-29%) while it was more stable against other European currencies such as the Swiss franc, the Swedish krona (both -6%) and it was unchanged against the Danish krone. After reaching a trough in October 2000, the euro bottomed out. Following a period of protracted weakness, the euro moved in 2002 onto a steady recovery track to trade by the end of 2004 well above the levels observed at its launch before reverting again somewhat in 2005.

The understanding of whether and when such wide fluctuations in the euro exchange rate constitute movements towards or away from some long-term equilibrium level is crucial from a policy perspective. Such an assessment is, however, complicated by various factors: Given its launch in 1999, the euro exchange rate has a rather short history. As a result, it is not straightforward to scrutinise the evolution of the euro over this period against longer term trends. In order to overcome this obstacle, it has become common practice to use a proxy measure for the euro for the period before its actual existence. In more detail, either the D-mark, which was de facto the anchor currency among the European currencies participating in the exchange rate mechanism, or a "synthetic" euro exchange rate, i. e. a weighted average of the euro legacy currencies has been commonly used. While Clostermann and Schnatz (2000) show that the evolution of the real synthetic euro-dollar exchange rate and the real D-mark-dollar exchange rate is very similar, more recently, Nautz and Offermanns (2006) provide empirical evidence that the synthetic euro exchange rate constitutes a better proxy for the euro prior to 1999 as compared to the D-mark.[2] Another problem in the assessment of the euro exchange rate relates to the appropriate equilibrium concept. While there are various ways to think about equilibrium exchange rates (see ECB, 2002), this paper focuses on the universal starting point of applied exchange rate analysis which is to compare the current exchange rate against some measure of purchasing power parity (PPP). In its relative version, PPP states that nominal exchange rates move in proportion to relative developments in domestic and foreign prices implying a constant real exchange rate.

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[2] For the construction of the synthetic euro exchange rate see Buldorini et al. (2002). For applications see for instance Alberola et al (1999), Chinn and Alquist (2001), Maeso-Fernandez et al. (2001).

Taylor and Taylor (2004) show in a recent review of the extensive empirical research on PPP that the recognition of this concept itself has gone through several boom and bust cycles and that its viability as an exchange rate benchmark is still a subject of heated controversy in academia. While it is widely accepted nowadays that the PPP relationship does not hold at every point in time, the more important issue is whether the PPP exchange rate indeed creates a ”centre of gravity” for real exchange rate developments in the medium to long run. Accordingly, for being an effective tool in the exchange rate analysis, at least a “weaker” version of the PPP hypothesis needs to hold, namely that the real exchange rate has a tendency to revert to its own long-run mean. Standard formal testing procedures – so-called unit root tests – commonly fail to confirm the presence of such mean reverting properties for real exchange rates. If a unit root would indeed mimic the true data generating process (DGP) of the real exchange rate, it would behave like a “random walk” without a systematic tendency to revert towards its PPP equilibrium.

In spite of the prevailing doubt about the usefulness of PPP benchmarks, Rogoff (1996) claimed about ten years ago that practitioners, while not taking PPP seriously as a short-term proposition, “… instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates”. Since then, the gap between theory and practice has narrowed. Significant progress has been made to better substantiate the validity of the PPP hypothesis. An important milestone in this context relates to the finding, that the commonly used statistical tests lack power. Based on Monte Carlo simulations, Sarno and Taylor (2002) convincingly demonstrate that it would require 200 years of data to correctly reject the unit root in real exchange rates in standard tests if the true DGP is indeed stationary with slow mean reversion. In fact, Taylor and Sarno (1998) employ very long data series and find some more evidence for at least slow mean reversion and, thus, the validity of relative PPP. Likewise, evidence based on panel data sets is more in favour of the PPP hypothesis (see, for instance, Anker (1999)).

Both kinds of evidence were subsequently subject to criticism, however: Very long time series are exposed to countless structural breaks as they encompass many regime shifts for the exchange rate, international trade and the underlying policy, in general. As regards panel data studies, O’Conell (1998) has argued that these findings strongly depend on the base currency, particularly, if cross-section dependence in the data is not properly controlled for. Moreover, Taylor and Sarno (1998) suggest that the usual evidence on PPP entail some interpretation problems as they only allow to conclude that at least one real exchange rate in the sample behaves in accordance with PPP, but they do not allow more general conclusions about the validity of the PPP theory. More recently, a study by Fischer and Porath (2006) presents evidence based on Monte Carlo experiments that panel unit root tests are severely biased in favour of rejecting the non-stationarity null if the real exchange rate contains MA roots.[3] The latter can be justified either by the presence of divergent productivity developments in the traded and non-traded sectors of the economy (i.e. the Balassa-Samuelson argument) as pointed out by Engel (2000), or by the decomposition of the real exchange rate into a stationary real exchange rate for a single good and the (non-stationary) weighted sum of relative prices of all other goods (see Fischer, 2006).

This paper contributes to the discussion by applying another promising research avenue to the *euro* exchange rate, namely that the crucial mean reversion properties of the PPP hypothesis could be subject to non-linearities. Evidence for such behaviour has been presented already for a number of other currency pairs. For instance, Sarno (2000a) as well as Taylor et al. (2001) examine the dollar exchange rates of the pound sterling, the yen, the D-mark and the French franc. Kilian and Taylor (2003) analyse in addition to these four exchange rates also the US dollar exchange rates of the Canadian dollar, the Italian lira and the Swiss franc. Their sample period, however, finishes in 1998Q4, i.e. before the launch of the euro. Sarantis (1999) finds non-linearities in real effective exchange rates based on unit labour costs of 10 OECD countries over the period 1980 to 1996. Baum et al. (2001) employ a two-step approach by first deriving deviations from PPP in a cointegration framework and assessing subsequently non-linear adjustment towards PPP for US dollar real exchange rates of up to 17 countries based on different price measures over the post Bretton Woods period. More recently, Reitz and Taylor (2006) analyse non-linear exchange rate behaviour in the context of foreign exchange market intervention for the D-mark vis-à-vis the US dollar. Non-linearites in the adjustment of exchange rates have been also tested for more exotic currencies. For instance, Sarno (2000b) examines real exchange rate behaviour in the Middle East and Liew et al. (2004) look into the non-linearities in the Singapore dollar-US dollar exchange rate. For the time being, however, this approach has not been rigorously applied to the euro exchange rate, although the euro is the second most actively traded currency in foreign exchange markets worldwide and the euro-dollar exchange rate is by far the most traded currency pair (BIS, 2005).[4]

This paper confirms the presence of non-linearities in the mean reversion for the real effective exchange rate of the euro and the two bilateral euro exchange rates with the biggest weight in the euro effective exchange rate, namely the real euro-dollar and the real euro-pound sterling

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3 This assessment builds on Engel (2000) who showed on the basis of Monte Carlo simulations for univariate unit root tests that those may be biased if favour of rejecting non-stationarity if the real exchange rate consists of a stationary and a non-stationary component and follows an ARIMA (1,1,1) process.

4 One exception is Lahtinen (2006), who uses a logistic smooth transition autoregressive model for the euro-dollar exchange rate, but actually employs the D-mark exchange rate before 1999. Moreover, the results suggest that the transition is discrete rather than smooth as one would expect from STAR-model.

exchange rates. The next section provides some stylised facts as well as possible economic rationales for the presence of non-linearities in the adjustment process of exchange rates. Thereafter, we offer some provisional statistical evidence based on survey data which suggests that PPP is a concept indeed taken into account by market participants when assessing the exchange rate level and that some features consistent with non-linearities in the adjustment process may be present. Subsequently, smooth transition autoregressive (STAR-) models are briefly presented as a method to identify non-linearities in exchange rates. Finally, we investigate whether the euro real exchange is subject to such non-linear behaviour in its mean reversion to PPP.

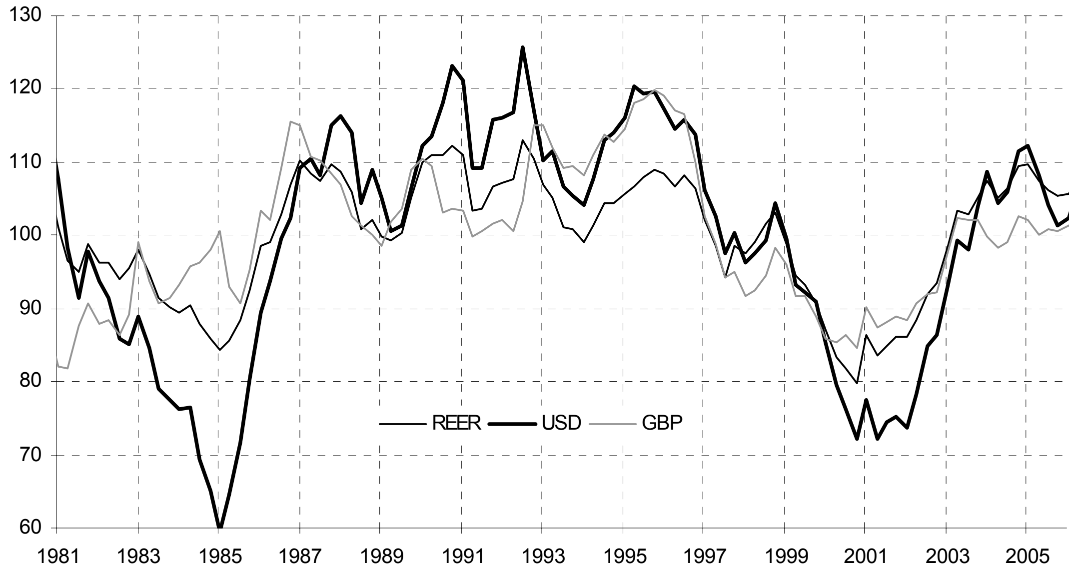
2. Stylised facts on euro real exchange rates

The real euro exchange rate is defined as nominal euro exchange rate (in terms of foreign currency per euro) multiplied by the domestic consumer price index and divided by the foreign consumer price index. Consumer prices have been used mainly for availability reasons, but the results should be robust to the use of different deflators as there is a strong correlation between the real euro exchange rates based on various cost and price measures over the past ten years.[5] Relative PPP implies that the real exchange rate oscillates around some long-term average level. Visual inspection of the evolution of the euro real exchange rates based on consumer price indices appears to support the idea that these rates have been subject to wide fluctuations around some mean level.

Consistent with the relative PPP concept, in Chart 1 the real euro-dollar, the real euro-pound sterling and the real effective exchange rate of the euro have been normalised so that the average level of each series over the sample period equals hundred. The euro real effective exchange rate, for instance, stood in late 2000 around 20% below its twenty-five year average, while it stood in the early 1990s more than 10% above this level. Fluctuations have been even stronger for the euro-dollar exchange rate which stood at its peak in the mid-1980s about 40% below its long-term average and in the first half of the 1990s by more than 20% above this average. Fluctuations in the pound-euro exchange rate were smaller over this period. In the long run, however, these time series seem to suggest that exchange rates appear to revert to some mean over the sample period covering 1980Q1–2006Q2.

[5] There are merits and drawback for using various cost and price measures in computing real exchange rates (see ECB 2002).

Chart 1: Real USD/EUR, GBP/EUR and euro effective exchange rate (EER-12)
(Indices: 1980-2005 = 100; CPI-deflated; quarterly data in logarithms)



Source: ECB.
Note: Last observation refers to the second quarter of 2006. The EER is computed against 12 important trading partners.

More rigorous standard statistical tests on the mean reversion characteristics of the real exchange rate, however, provide a more controversial picture. Both the Augmented Dickey-Fuller (ADF-) test and Philips-Peron (PP-) test commonly fail to confirm the presence of such mean reverting properties for euro exchange rates (see Table 1). From this testing perspective, therefore, the evidence for mean reversion remains scarce, also for euro exchange rates.

Table 1: Tests for mean reversion

	ADF-test (p-values)	PP-test (p-values)
EER	0.168	0.137
USD	0.162	0.133
GBP	0.113	0.206

Augmented Dickey-Fuller (ADF-) test including a constant term and lags selected according to the Schwartz criteria. Philipps-Peron (PP-) test is an nonparametric alternative to the ADF-test. Both tests have the null hypothesis that the underlying time series is non-stationary (unit root process). Critical values as provided by MacKinnon (1991) are applied.

A major drawback of this procedure is, however, that these tests assume that the speed of equilibrium correction is the same whether the exchange rate is close to its PPP equilibrium level or far off. If the mean reversion properties were in fact non-linear, however, these tests would be mis-specified, which could explain this standard findings that they fail to support PPP for euro real exchange rates as a long-run equilibrium concept.

3. The rationale behind non-linearities in real exchange rates

There are sensible economic reasons why the real exchange rate could behave in such non-linear fashion:[6] From a goods market perspective, PPP in its strict form assumes perfect inter-country commodity arbitrage. However, international arbitrage is subject to market frictions and transactions costs including sunk costs for entering (or exiting) a market. This creates an implicit "inner band of inaction" around the real exchange rate where arbitrage is simply too costly and, thus, does not take place. However, once the real exchange rate moves for a sufficiently long period outside some upper or the lower thresholds, arbitrage processes set in and return the real exchange rate towards its long-term mean (until the limits of the band of inaction is reached again). This indeed suggests that the real exchange rate should become increasingly mean-reverting with the size of the deviation from the equilibrium level (see Taylor et al. 2001).

Taking more an asset market perspective, exchange rate behaviour might be affected by heterogeneous foreign exchange traders: "Chartists" and "Fundamentalists". If the exchange rate is close to its equilibrium level, "Chartists" dominate the market as the exchange rate dynamics is mainly driven by "technical analysis". In such circumstances, the exchange rate behaves like a "random walk" and the PPP equilibrium as an attractor is very weak. However, as the exchange rate moves increasingly away from its perceived equilibrium and consensus builds up that the exchange rate is indeed misaligned, exchange rate analysis focussing on "fundamentals" gains importance and the so-called "Fundamentalists" drive the exchange rate back towards its equilibrium. In this context, Killian and Taylor (2003) also emphasise the degree of uncertainty in exchange rate modelling and the associated role played by the difficulty to determine the "correct" exchange rate equilibrium. Even if the PPP relationship would be generally accepted as a valid equilibrium concept, the empirical determination of such equilibrium would depend on the base period chosen, the sample period and/or the underlying cost and price concept. For the effective exchange rate, the methodology for determining the weighting scheme and the chosen group of trading partners are additional relevant factors. The controversy about the correct equilibrium exchange rate level increases further if fundamentals-based models are employed. However, as the exchange rate becomes

[6] See in particular Dumas (1992) for a theoretical justification for non-linearities in exchange rates based on a general equilibrium model.

increasingly over- or undervalued, consensus among the models employed and foreign exchange market traders is likely to rise and, thus, the heterogeneity of beliefs of the traders declines and speculation towards equilibrium resumes. By contrast, if the exchange rate is close to some "latent" equilibrium, there is no consensus as to whether the exchange rate is over- or undervalued and the exchange rate is driven mainly by noise traders.

Both the goods and the asset market perspectives imply that mean reversion takes place only once the exchange rate is sufficiently far away from its equilibrium. The next section presents some evidence based on survey data that such factors may be at work. Subsequently, an econometric approach is employed to detect non-linearities in the adjustment of euro exchange rates.

4. Evidence for non-linearities for in euro exchange rates

4.1. Survey evidence for the euro-dollar exchange rate

For the euro-dollar exchange rate, survey data indeed provides some tentative evidence that market participants' views on exchange rates become more homogeneous as the exchange rate moves far away from its PPP equilibrium. The World Economic Survey conducted quarterly by the ifo Institute for Economic Research asks 314 participants about their expectation of US dollar developments over the next six months.[7] The survey encompasses mainly managers from the relevant divisions of corporates, banks and insurance companies, and it also takes into account the opinions of academics and applied economists. The coverage of the survey appears to be more comprehensive than other forecast products available. Chart 2 shows the evolution of the deviation of the euro-dollar exchange rate from its PPP level and the ifo survey indicator. A negative survey indicator signals expectations of a euro appreciation.

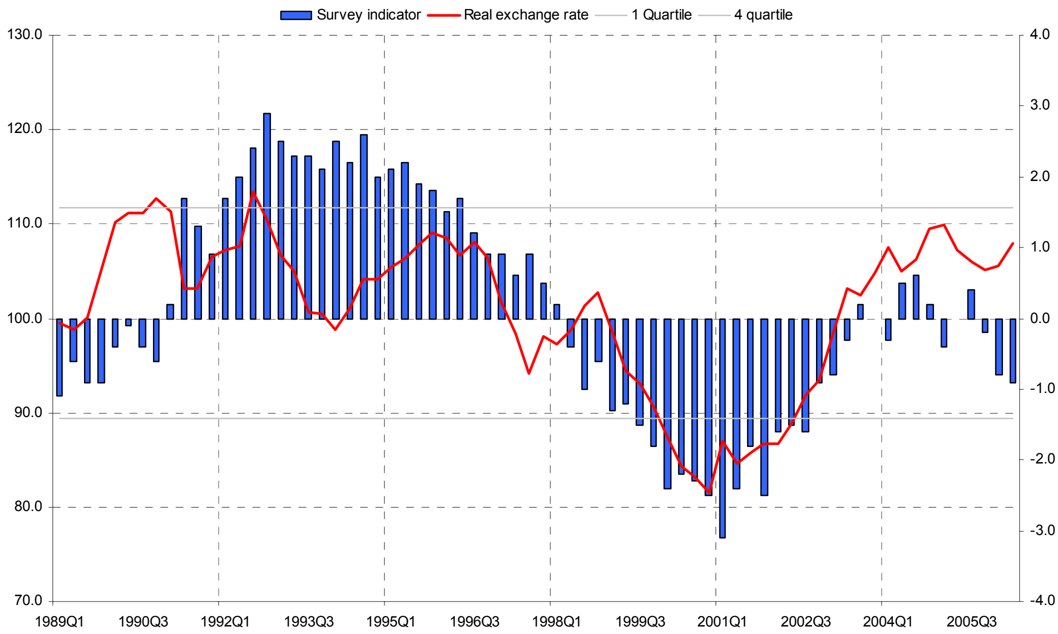
The Chart highlights two important results: Firstly, there is a rather strong co-movement between the survey indicator and the PPP misalignment for the euro-dollar exchange rate, suggesting that the PPP equilibrium constitutes indeed an important benchmark which market participants take into account when forming a view on future exchange rates developments. Secondly, in episodes when the exchange rate is closer to equilibrium, the opinions about future developments of the euro-dollar exchange rate are less unanimous than in periods of

[7] The ifo institutes asks for the value of the USD in relation to a country's currency by the end of the next six months. Accordingly, a corresponding assessment on other euro exchange rates is not directly possible. Prior to euro adoption, the ifo employs a weighted average of the euro legacy currencies.

large deviations. This is fully consistent with the rationale for non-linearities in exchange rates as outlined above.

Chart 2. PPP deviation of the real USD/EUR exchange rate and market expectations

(Left-hand scale: Real exchange rate: index 1980-2006 = 100; Right hand scale: ifo survey index: Score index range from -4 to +4, quarterly data)).



Source: ECB, ifo.

Note: A rise in the real exchange rate reflects a real euro appreciation. A negative ifo index signals appreciation expectations for the euro against the US dollar (over the next six months). Last observation refers to 2006Q2.

The distribution of the real exchange rate deviation from its long-term average provides more evidence in support of this hypothesis. Since 1989, in the extreme quartiles, the direction of the misalignment was associated with expectations that the misalignment will be (partially) corrected over the next six months in 94% of the cases. Only in the second half of 1990, an "undervaluation" of the US dollar was associated with expectations for a further decline in the US dollar. By contrast, respondent's expectations have been more heterogeneous in the two quartiles in the middle: In this range, only in 63% of the observations the respondents expected a correction of the PPP deviation while 37% forecast a further expansion of the PPP deviation. Since the launch of the euro, there was no signal in the extreme quartiles inconsistent with PPP, while there was still a 35% probability that participants predicted a change in the exchange rate which would raise the magnitude of the PPP deviation in the middle quartiles.

Using instead of the ifo survey data forecasts on exchange rates provided by Consensus Economics suggests a less prominent – albeit still present – role for PPP. The Consensus Economics data has the advantage that it surveys banks which are likely to be closer to professional foreign exchange trading. However, the number of respondents is smaller and the replies may also reflect strategic behaviour as market players may have stronger incentives not to reveal *ex ante* their true expectations. In episodes of extreme deviations from PPP, a change in the exchange rate towards the equilibrium is suggested in 80% of the cases (since the launch of the euro in 1999), while in episodes when the exchange rate was closer to its PPP level, the probability that market participants expected a change in the exchange rate towards the PPP level was about as high as expecting a move away from equilibrium.

4.2. Econometric evidence

Econometrically, exchange rate behaviour resulting from such trading patterns could be modelled as an inner and an outer regime for the real exchange rate, which are separated by a certain threshold. For PPP to hold, one would expect mean reversion in the outer regime while in the inner regime, the exchange rate would move randomly. Such a discrete switching may be appropriate when looking within a two-country framework only at identical goods with constant and proportional transaction cost. Relaxing these rather strict assumptions would argue for a smooth – instead of a discrete – transition from one regime to the next. In comparison to the econometrically convenient approach, which assumes a uniform adjustment speed at all times and for all magnitudes of deviation, these arguments would suggest a speed of convergence to the equilibrium to gradually increase as the deviation from equilibrium rises in absolute value.

As demonstrated by Taylor et al. (2001), among others, so-called smooth transition autoregressive (STAR) models (Granger and Teräsvirta, 1993 and Teräsvirta, 1994) are well suited to embody such dynamics for the exchange rate. The strength of the equilibrating force is increasing in the absolute magnitude of the degree of disequilibrium. In principle, STAR models are threshold autoregressive models (TAR) with an infinite number of regimes and a continuously varying and bounded adjustment speed. The transition function is continuous and non-decreasing. In addition, there are different alternatives of STAR-models. Most importantly, the exponential STAR-model (ESTAR) has the salient feature to be symmetric, which appears to be the most appropriate modelling strategy for non-linear real exchange rate dynamics. This is the starting point but it is also tested against the alternative of a logistic STAR-model which would be characterised by asymmetries in the mean reversion.

Following Teräsvirta (1994), the first step in the ESTAR model requires determining the appropriate linear autoregressive model based on the partial autocorrelation function (PACF). For each of the three exchange rates under consideration, this was found to be p=2. The second step in such a procedure involves testing for the presence of non-linearities in the adjustment of (the logarithm of) the real exchange rate (q) using the following relationship (see Sarno (2000a)):

$$q_t = \alpha_1 + \alpha_2 q_{t-1} + \alpha_3 q_{t-2} + \alpha_4 q_{t-1} q_{t-d} + \alpha_5 q_{t-2} q_{t-d} + \alpha_6 q_{t-1} q_{t-d}^2 + \alpha_7 q_{t-2} q_{t-d}^2 \quad (1)$$

where $q_t = s_t + p_t - p_t^*$, with s_t being the logarithm of the nominal spot exchange rate (the foreign price of domestic currency), p_t is the logarithm of the domestic consumer price level and p_t^* is the logarithm of the foreign price level. The data is normalised to the average level over the sample period so as to simplify the STAR specification below. Rejecting that the non-linear terms α_4 to α_7 are (jointly) zero would support the presence of remaining non-linearities in the exchange rate dynamics. The regression also includes the so-called delay parameter, d, which suggests that the rise in mean reversion is generated only with a delay. As this delay is unknown, it is commonly tested for a series of plausible values. The one with the lowest p-value for the hypothesis test is typically chosen. Following Sarno (2000a), we included up to six delays in the data set. We find clear evidence for remaining non-linearities for the real euro-dollar and the real effective euro exchange rate at the 5% level of significance using a delay of one year and evidence for the pound sterling almost at the 5% level. (see Table 2).[8]

Table 2: Test for remaining non-linearities in PPP equations (p-values)
Lowest marginal significance level in bold

d	EER p=2	USD p=2	GBP p=2
1	0.0768	0.0423	0.8202
2	0.0399	0.0531	0.3038
3	0.1076	0.0067	**0.0653**
4	**0.0268**	**0.0014**	0.1080
5	0.2139	0.0126	0.1622
6	0.9410	0.1349	0.4964
LSTAR vs. ESTAR	*0.07*	*0.32*	*0.80*

In order to identify possible asymmetries in the non-linear adjustment process, cubic terms have been added to equation (1). If the null hypothesis that these terms are zero was rejected, a logistic smooth transition model (LSTAR) would be the more appropriate specification. For the euro-dollar rate and for the euro-pound rate, the null hypothesis of no asymmetries cannot be rejected at the standard confidence levels. For the euro effective exchange rate, the null is rejected at the 10% level but not at the 5% level. For alternative significant delay parameters (d=1,2) in the specification using the EER, however, there is no evidence in favour of

8 An ordinary F-test is used to approximate the LM-test as it has relatively good size and power properties in finite samples.

asymmetric non-linearities. Accordingly, it is assumed in the following that asymmetries are not present STAR specifications.

Given this evidence, the reparametrized ESTAR model of the following form is estimated using non-linear least square (NLS-) methods, which still nests the standard linear regression autoregressive model (see Sarno 2000a, Baum et al. 2001):

$$\begin{aligned}\Delta q_t = \quad & \alpha_1 + \alpha_2 q_{t-1} + \alpha_3 \Delta q_{t-1} \\ & + \left[\alpha_4 + \alpha_5 q_{t-1} + \alpha_6 \Delta q_{t-1}\right]\left[1 - \exp(\alpha_7 (q_{t-d})^2)\right] + \varepsilon_t \end{aligned} \tag{2}$$

The first line of the equation reflects the "inner regime" as represented by the standard ADF-test for non-stationarity (assuming two autoregressive lags in levels). The second line represents the "outer regime" with the non-linear adjustment term. The second term in parenthesis is the so-called transition function, which is U-shaped and symmetric in this ESTAR formulation and bounded between zero and one.[9] ε_t is a random disturbance term. The transition parameter α_7 determines the speed of transition between the two regimes with a lower absolute value implying slower transition. In the presence of no non-linearities in the process ($\alpha_7 = 0$), the second term is zero and the model reverts to a linear specification corresponding to the standard ADF-model. However, if model (2) is the true specification, then the standard ADF model would be misspecified and its parameters would be estimated inconsistent as it would estimate the parameter α_2 as a combination of α_2 and α_5 of the true model. In the ESTAR model, by contrast, the coefficient α_2 will govern the adjustment process for small deviations from PPP, which may indeed represent random walk characteristics, while the transition function converges towards unity when the deviations from PPP are large.

When estimating equation (2), the delay parameters are set as suggested by the lowest p-values in Table 2. As the data has been demeaned and the real exchange rate series do not indicate the presence of a deterministic trend, the constant terms (α_1, α_4) in equation (2) should be equal to zero. Further standard restrictions are $\alpha_3 =- \alpha_6$ and $\alpha_5= -(1+ \alpha_2)$. However, as α_2 is likely to have random walk properties, it can be hypothesised that $\alpha_2 = 0$ and $\alpha_5 = -1$. After successfully testing and imposing these restrictions, we find evidence for non-linearities for the three exchange rates (see Table 3). For each exchange rate, the transition parameter is negative and significantly different from zero at standard levels for the euro-dollar and the EER, while it is more at the margin for the pound exchange rate, if the critical values of MacKinnon for ADF-tests were employed.

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[9] For a more general discussion on the form of the transition function see Teräsvirta (1994).

Table 3: Estimation results on non-linear mean reversion

	EER	USD	GBP
	p=2, d=4	p=2, d=4	p=2, d=3
Mean-reversion coefficient (α_7)	-6.43	-1.82	-4.53
	(2.59)	(2.88)	(2.20)
	[0.0015]	[0.0006]	[0.0108]
Autoregressive lag ($\alpha_3 = \alpha_6$)	0.28	0.26	0.25
	(2.82)	(2.91)	(2.61)
Diagnostics (p-values):			
AR(1)	1.00	1.00	0.51
AR(4)	0.93	0.92	0.19
ARCH(1)	0.28	0.44	0.11
ARCH(4)	0.40	0.92	0.44
JB	0.54	0.39	0.61

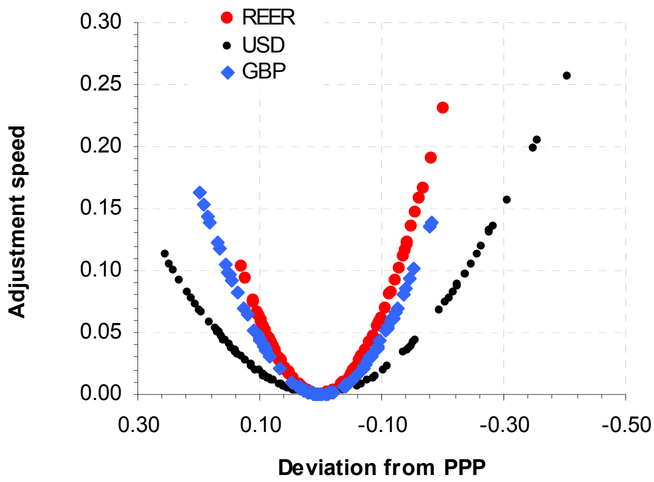
Note: Numbers in parenthesis are t-values, numbers in squared brackets are simulated p-values.

Since Sarno (2000a) as well as Kilian and Taylor (2003) suggest that such t-ratios need to be interpreted with caution in the presence of a unit root under the null hypothesis, empirical marginal significance levels were also computed by Monte Carlo methods assuming that the true data generating process for (the logarithm of) each real exchange rate was a driftless random walk. The empirical significance levels were based on 20,000 simulations of the length 600 from which the first 500 data points were discarded. At each replication an ESTAR specification in the form of (2) was estimated. The percentage of replications for which a t-ratio for the estimated transition parameters greater in absolute value than the one reported in Table 3 was taken as the empirical significance level in each case. The corresponding p-values were included in square brackets below the estimated t-values. These results further strengthen the evidence of non-linear mean reversion for the real euro exchange rates under consideration over the past 25 years. The diagnostic statistics do not suggest remaining autocorrelation or heteroskedasdicity in the errors, but it seems that this methodology does not purge all non-linearities.[10]

Chart 3 illustrates the mean-reverting properties of the various real euro exchange rates. It shows that the speed of mean reversion (vertical axis) – and, thus, the strength of the gravity as determined by the so-called transition function – increases non-linearly with the magnitude of the (lagged) deviation from the PPP equilibrium (horizontal axis).

[10] The test proposed by Eitrheim and Teräsvirta (1996) cannot reject the null of no remaining non-linearities in the residuals for certain reasonable values for delay parameters for the pound sterling. However, it cannot be ruled out that a more complex non-linear structure is present in the ESTAR equation for the real effective exchange rate (at the 10% level) and for the euro-dollar exchange rate (at the 5% level).

Chart 3. Speed of adjustment and PPP deviation.



Source: Author's calculations.

It provides several additional insights: *Firstly*, the functions for the euro-dollar exchange rate and the EER are skewed towards negative territory, reflecting that these exchange rates experienced more extended downward deviations from PPP than upward deviations. *Secondly*, for the euro-dollar, deviations from the PPP equilibrium need to be stronger than for the other exchange rates in order to reach the same re-adjustment force. This is consistent with relatively stronger fluctuations of the euro-dollar exchange rate than in the other real exchange rates. *Thirdly*, these functions suggest that the speed of reversion is commonly relatively low. For the effective exchange rate, for instance, a movement of the real exchange rate by 10% away from its long-term average would raise the transition function to just about 0.065. To get a sense about the speed of mean reversion, it is usually measured how long it would take for the effect of a shock to die out: the so-called half-life of a shock. In the present case, the estimated coefficient would imply a half-life of such a shock of more than 2½ years. The mean of the adjustment speed is for all currencies at around 0.04, which corresponds to a half-life of 4-4½ years (see Table 4). Intriguingly, this result is fully consistent with the prominent finding of Rogoff (1996), that the ("average") adjustment process to PPP following a shock has a half-life of three to five years.

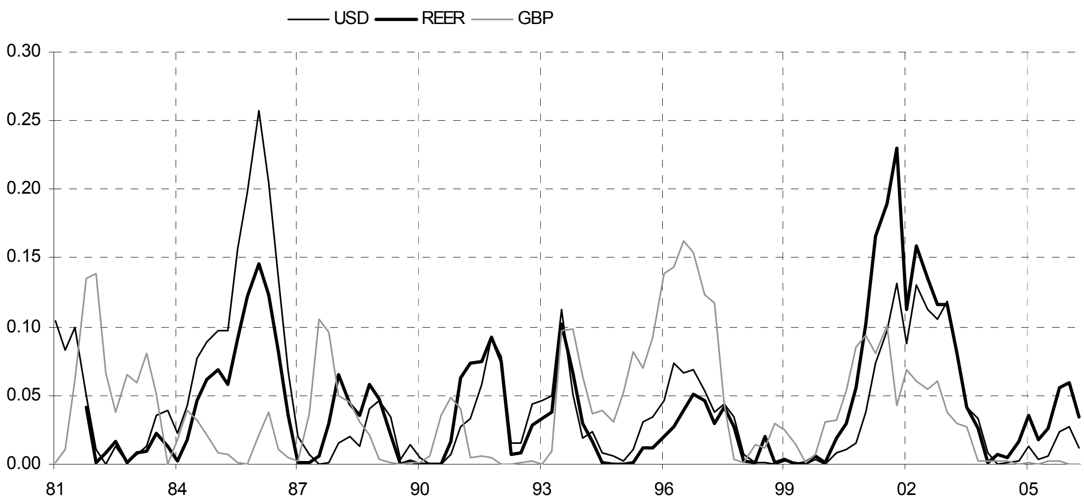
However, in episodes when the real exchange rate has moved far away from its long-term average, the forces of gravity become gradually stronger. For the real EER, for instance, shortly after the euro had experienced a historical low in the fourth quarter of 2000, the transition function gradually increased and reached a peak in the fourth quarter of 2001. This coincided already with some appreciation of the euro as the deviation from PPP declined from 20% to 13%. At this point in time, the model suggests that the adjustment intensity was much stronger implying a half-life of the deviation of only about two to three quarters of a year. Broadly consistent with the model, the deviation from PPP equilibrium was halved over the next three to four quarters as the euro experienced a strong recovery in 2002.

Table 4 Speed of adjustment in different episodes

	EER	USD	GBP
Mean of the transition function	0.042	0.043	0.038
Half-life in quarters (average)	16.2	15.6	17.9
Peak of the transition function	0.2302	0.2571	0.1626
Date of the peak	2001Q4	1986Q1	1996Q3
Half-life in quarters (minimum)	2.6	2.3	3.9

In this episode, the local maximum reached for the euro-dollar in the transition function was lower than for the real EER (see Chart 4).

Chart 4. Adjustment speed and PPP



Source: Auhtor's calculations.

However, it also reached a peak at 0.13, implying that the gravity forces were much stronger in 2000/2001 than in normal periods. For the US currency, the mid-1980s constitute another interesting episode. At its peak in the first quarter of 1986, when the US dollar correction was in full swing, mean reversion properties suggested a much shorter half-life of PPP deviations.[11] At the end of the sample period, all the real euro exchange rates under review have been reasonably close to their long-term average (see Charts 3). As a result, the PPP mean-reverting forces were rather dormant, as indicated by the transition functions being close to the zero line.

5. Conclusion

Summing up, this paper suggests that – as a starting point – the PPP equilibrium provides a useful guide for assessing the euro exchange rate and its potential medium-term course, particularly if the real exchange rate deviates strongly from its long-term average. We show on the basis of survey data that market participants may be uncertain about the equilibrium exchange rate if it is close to its PPP level but consensus emerges as the magnitude of the deviation from the PPP equilibrium rises. Econometrically, the analysis showed that the mean-reverting properties strengthen non-linearly with the deviation from the PPP equilibrium: These forces are weak in periods when the exchange rate is close to its long-term average. Accordingly, this indicator does not seem to be a very effective tool entering a forward-looking exchange rate assessment in such episodes. In periods of significant deviations from its long-term average, by contrast, gravitational forces are set to take root and bring the exchange rate back towards its long-term trend.

Looking ahead, the issue of remaining non-linearities should be addressed in future research by analysing an additive STAR model as an alternative to the present specification. Moreover, it could be useful to combine non-linearities with the literature on fundamentals-based equilibrium exchange rate modelling. The latter is commonly modelled in a linear vector-error correction framework which takes the interaction between the (real) exchange rate and its driving factors into account. Some initial research in this direction involving the euro-dollar exchange rate has been presented recently by Korhonen (2005), but he rightly states that the statistical theory has not been fully developed yet.

[11] The quantification of the speed of adjustment derived from ESTAR models require some caution. Paya and Peel (2006) recently showed via simulation and bootstrap methods, that such estimates may be upward biased in typical sample sizes.

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